Fifty-fourth Annual Allerton Conference
Allerton House, UIUC, Illinois, USA
September 27 - 30, 2016

Modeling the P300-based Brain-computer Interface as a Channel with Memory

Vaishakhi Mayya*, Boyla Mainsah*, and Galen Reeves*†
*Department of Electrical and Computer Engineering, Duke University, Durham, NC, USA
†Department of Statistical Science, Duke University, Durham, NC, USA

***Abstract*—The P300 speller is a brain-computer interface that enables people with severe neuromuscular disorders to communicate. It is based on eliciting and detecting event-related potentials (ERP) in electroencephalography (EEG) measurements, in response to rare target stimulus events. One of the challenges to fast and reliable communication is the fact that the P300-based ERP has a refractory period that induces temporal dependence in the user's EEG responses. Refractory effects negatively affects the performance of the speller. The contribution of this paper is to provide a model for the P300 speller as a communication process with memory to account for refractory effects. Using this model, we design codebooks that maximize the mutual information rate between the user's desired characters and the measured EEG responses to the stimulus events. We show simulation results that compare our codebook with other codebooks described in literature.**

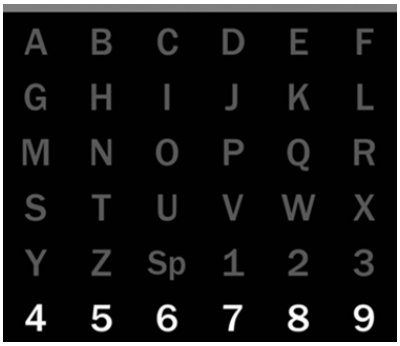


Fig. 1: P300 speller visual interface. The bottom row has been illuminated ('flashed') in this example

I. Introduction

A brain-computer interface (BCI) is a system that monitors electrophysiological signals and translates the information encoded in these signals into commands that are relayed to a computer that carries out an action [1]. BCIs provide a communication alternative for individuals with severe neuromuscular diseases that impair neural pathways that control muscles [2], [3]. In the extreme case of locked-in syndrome, individuals lose all voluntary muscle control, and have very limited ability to communicate verbally or via gestures. However, these individuals still retain the cognitive function necessary to control BCI systems. The P300 speller, developed by Farwell and Donchin [4], is a BCI that has been used to help individuals with severe neuromuscular disabilities to communicate, such as those with amytrophic lateral sclerosis [2], [3].

The P300 speller relies predominantly on eliciting and detecting event related potentials (ERP) embedded in electroencephalography (EEG) data. These ERPs are elicited in response to specific stimulus events within the context of the *oddball* paradigm [5]. In the oddball paradigm, a user is presented with a random sequence of stimulus events that fall into one of two classes: a rarely occurring oddball or target stimulus and more frequently occurring or non-target stimulus [5]. The presentation of the rare target stimulus event elicits an ERP response, which includes a distinct positive deflection called the P300 signal. In the P300 speller, the goal is to enable the user to communicate, by spelling words one letter at at time.

In a visual P300 speller, a user a presented with an array of choices on a screen, such as the grid as shown in Figure 1. To communicate a given character, the user focuses on that character on the screen. Subsets of characters or flash groups are sequentially illuminated on the grid. In this context, the illumination of a flash group is a stimulus event. Ideally, if user's intended target character is a part of the flash group, the P300 ERP is elicited. Following each stimulus event, a time window of the EEG waveform is analyzed to calculate the likelihood that the stimulus event contains the target. However, the elicited ERPs are embeded in noisy EEG data, which makes detection challenging due to the low signal-to-noise ratio of the elicited ERPs. For improved accuracy, the target character is identified by repeating the process several times with different flash groups, and selecting the character with the largest cumulative response after data collection.

The order of presentation of non-target and target stimulus events plays a significant role in the ERP elicitation process. This is due, in part, to refractory effects [6], where the ability to elicit a strong ERP response to every target stimulus event presentation is affected by the time between target stimulus events, called the target-to-target interval (TTI). If a P300 ERP is elicited following a presentation of a target, it is highly likely that the amplitude of a successive P300 ERP elicited in response to subsequent target events with a low TTI may be attenuated or distorted [7]. Refractory effects can misclassification of target stimulus events. The actual length of the refractory period is not known, and could possibly vary across different users and different types of systems [7].

The goal of this paper is to increase the rate of reliable

communication using the P300 speller by understanding the limits imposed by the refractory period. We propose a model for ERP elicitation where refractory effects are explicitly considered, by representing the P300 speller as a noisy communication channel with memory. Using this model, we compute the information capacity of the channel with memory for Markov sources. We compute the optimum distribution across the channel input that achieves this capacity using a generalization of the Blahut-Arimoto algorithm [8]. We use this optimum distribution to design flash groups for the P300 speller that accounts for refractory effects.

The rest of this document is organized as follows. Section II reviews the previous approaches to designing flash groups for the P300 speller and provides relevant background concerning the information rates for channels with memory. Section III introduces our new channel model and a method to design flash groups using this model and Section IV shows the preliminary results from simulations of P300 spelling runs. Section V includes discussion of the results and Section VI outlines future work on our model.

Notation: Random values are denoted by uppercase letters and their realizations are lowercase letters. The time index is $n \in \mathbb{Z}$ and $X_1^n = [X_1, \dots X_n]$ represents vectors of length n.

II. Background

A. P300 Channel and Codebook Design

There are several methods that could be used to describe the communication channel. We are interested in studying refractory effects and errors in estimating single characters due to a noisy EEG system. We consider the following model for the channel, Figure 3.

1) Source: The source is the character the user wants to transmit, drawn from a finite alphabet $w \in \{1, 2, \dots W\}$ such as the character choices in the grid shown in Figure 1.

2) Encoder: The encoder maps the W source symbols to a length-N binary codeword, X_1^N. The set of flash groups can be represented as a binary $W \times N$ codebook $\mathbf{C}$, where W is the number of characters, and N is the number of flash groups. In the context of the codebook, N can also be said to be the length of codeword associated with each character. Each row in the codebook corresponds to a character and the columns of the codebook correspond to the flash groups at a given sequence index n. If the symbol $w \in [1, 2 \dots W]$ is present in the flash group at time n, $\mathbf{C}(w, n) = 1$, else, $\mathbf{C}(w, n) = 0$. The code rate is $\frac{1}{N}$. If the target character is $w \in \{1, 2, \dots W\}$, the output of the channel encoder (input to the channel) for the n-th use of the channel is given by

$$X_n = \mathbf{C}(w, n).$$

Some codebooks that are currently used for the P300 speller are discussed later in this section.

3) Channel: The channel describes a probabilistic mapping from the input sequence X_1^N to the output Y_1^N. The connection between the BCI and an information channel has been considered previously. For example, Omar et al. [9] represent the BCI-based communication (based on motor imagery) as a memoryless binary symmetric channel. In the P300 speller, each stimulus event elicits a different response depending on whether it is or is not a target stimulus event, and the responses are embedded in noisy EEG data.

4) Receiver and Decoder: The EEG data is collected from multiple electrodes on the scalp. For each flash group, features relevant to the ERP components are extracted from the EEG data and scored using a classifier. The observed sequence Y_1^N at the decoder is either the vector of classifier scores, or the binary values associated with the presence or absence of the P300 obtained by thresholding the classifier score. A decoding function is used to select the most likely input sequence, X_1^N, given the received sequence Y_1^N.

The codebook design process is an important part of the P300-based communication system. Poor codebook design decreases the rate of communication. In particular, the order and timing of target stimulus events determines the relative degree with which refractory effects negatively impact performance

For codebooks designed by the row-column paradigm (RCP), the flash groups are the rows and columns of characters arranged in a grid layout, such as shown in Figure 1. [4]. The flash groups in the RCP are randomly presented, without replacement. An instantiation of codebook for RCP represented as a matrix is shown in Figure 2a. Due to the randomized order of presentation of row and column flash groups in RCP, it is likely that a character is flashed twice consecutively. In the Figure 2a, at both $n = 1$ and $n = 2$ the character 'X' is flashed, since the fourth row and last column of the grid is flashed successivley.

Successive target character presentation can be avoided by flashing a single character at a time [12]. However, single character presentation increases the time to communicate [13]. The checkerboard paradigm (CBP) [10], seen in Figure 2b, was developed to mitigate refractory effects by imposing a minimum interval between target character presentations. This also leads to sparse codewords, as illustrated in the CBP codebook example.

Other approaches to stimulus paradigm design have focused on designing flash patterns or codebooks that maximize the measure of mutual information between the stimulus events and the elicited EEG responses, e.g. [11], [14]. However, in real-time studies, the codebooks developed in [11], [14] resulted in similar or worse performance when compared to the RCP, due to limited consideration of P300 refractory effects. One of the codebooks generated by this approach, the D10 codebook, is shown in Figure 2c. The D10 codebook is characterized by long streams of repetitive character presentations, which increases the negative impact of refractory effects on performance.

Previous approaches for designing the stimulus presentation paradigm have focused predominantly on minimizing refractory effects by imposing a long TTI or optimizing the encoding process with a memoryless channel assumption. We

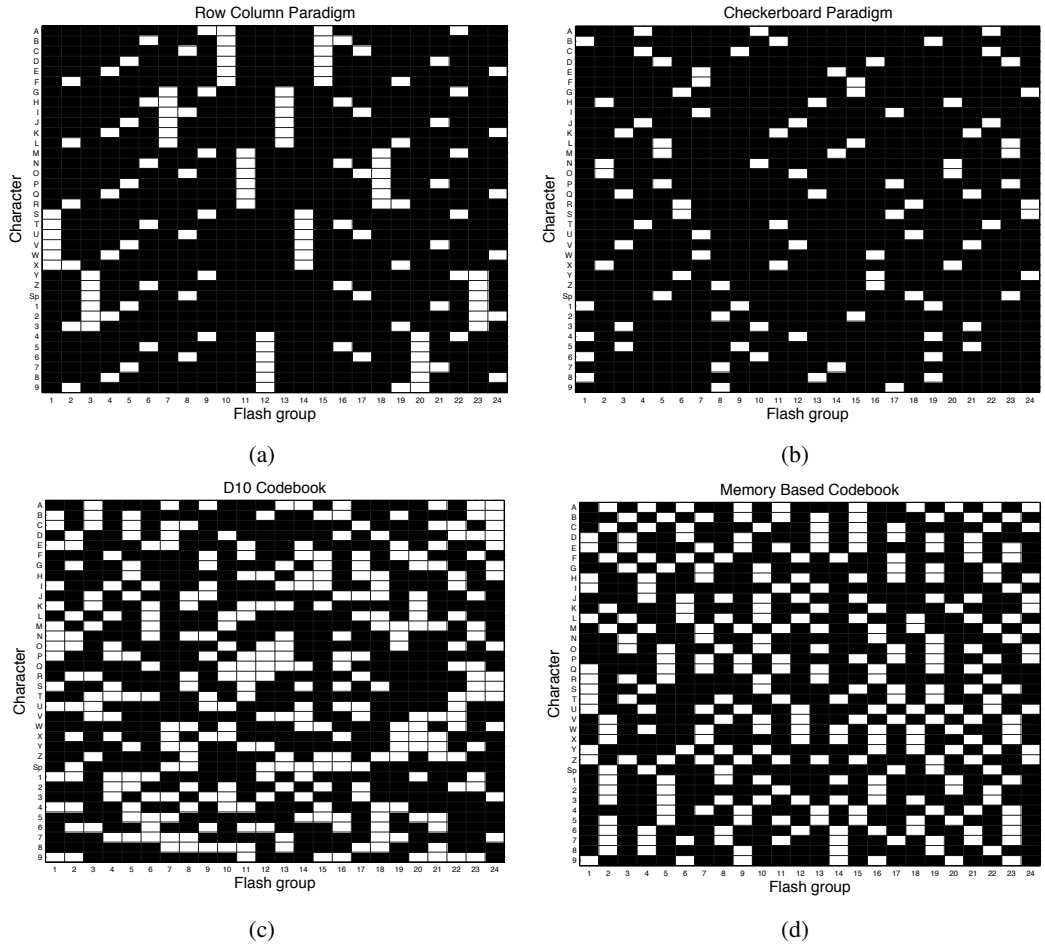


Fig. 2: Codebooks for different stimulus presentation paradigms based on the 6×6 P300 speller grid shown in Figure 1. Each column of the codebooks represents a flash group. Each row represents the codeword for a character. (a) Row-column paradigm (RCP) [4], where the flash groups are the rows and columns of a grid, presented in random order without replacement (b) Checkerboard paradigm (CBP) [10], where the flash groups are created from to virtual matrices of a checkerboard overlay of a grid (c) D10 paradigm [11], where the flash groups are designed based on maximizing the Hamming distance between codewords (d) The memory-based codebook is the codebook generated for our model of the channel with memory (Section III-B).

are interested in studying refractory effects and minimizing decoding errors due to a noisy EEG system, which requires a more complex channel model to account for the memory in the system

B. Capacity of Channels with Memory

In a channel with memory, the current channel output depends on the current and previous channel inputs and previous outputs. Let $X_n \in \mathcal{X}, n \in \mathbb{Z}$ where $\mathcal{X}$ is a finite alphabet for the input. The output, Y_n, is finite or continuous depending on the noise in the channel.

For our analysis, we represent channels with memory as finite state channels (FSC). FSCs were first introduced by Gallager [15]. The state of the channel encodes information about the previous inputs and other factors that affect the channel parameters such as noise. The state is represented by the finite set $S_n \in [1, 2, 3 \ldots L], n \in \mathbb{Z}$. FSCs are described by the conditional probability $P(Y_n, S_n | X_n, S_{n-1})$, the current state and output depend on the previous state and current input. We focus on a special class of FSCs, where the output and state sequence are statistically independent,

$$P(Y_n, S_n | X_n, S_{n-1}) = P(S_n | X_n, S_{n-1}) P(Y_n | X_n, S_{n-1}). \tag{1}$$

An FSC is said to be indecomposable if for any starting state $S_0 \in [1, 2, 3 \ldots L]$ and any ending state, S_n, it is possible to find an input sequence X_1^n such that $P(S_n | X_1^n, S_0) > 0$ [15]. In an indecomposable channel, every state of the channel is accessible from every other state.

An FSC is said to be an intersymbol interference (ISI) channel if the state and the output of the channel depend on current and previous inputs. For these types of channels,

it is possible to describe a state sequence that is uniquely determined by the input sequence, i.e. $P(Y_n|X_n, S_{n-1}) = P(Y_n|S_n, S_{n-1})$. It is also possible to represent the channel output as a hidden Markov process controlled by the state sequence $S_{-\infty}^n$.

We are interested in finding the capacity of FSCs and the distribution across the input that achieves capacity. We consider the input, X_1^N to be an r-th order Markov source, such that, $P(X_{n+1}|X_{-\infty}^{n-1}) = P(X_n|X_{n-r}^{n-1})$.

Let $P_r = P(X_{n-r+1}^n|X_{n-r}^{n-1}) = P(X_n|X_{n-r}^{n-1})$ represent the transition matrix for an r-th order Markov source. The information rate of an ISI channel is given by [8],

$$\mathcal{I}(P_r) = \lim_{N\to\infty} \frac{1}{N} I(S^N; Y^N|s_0), \tag{2}$$

where the initial state s_0 can be chosen arbitrarily. The capacity of the channel for the r-th order Markov source is given by

$$\mathcal{C}_r \coloneqq \sup_{P_r} \mathcal{I}(P_r). \tag{3}$$

The optimum input transition matrix that achieves the maximum information rate, P_r^*, is in the space of all $|\mathcal{X}|^r \times |\mathcal{X}|^r$ matrices. The entries of the transition matrix are between zero and one and the rows sum to one.

The sequence of maximum information rates, $\mathcal{C}_r$ is non-decreasing in r and forms lower bounds to the true capacity, $\mathcal{C} = \sup_{P_X} \mathcal{I}(X;Y)$ of the channel [16],

$$\mathcal{C}_1 \leq \mathcal{C}_2 \leq \ldots \leq \mathcal{C}_\infty = \mathcal{C}.$$

For a given r, the goal is to find the tranisition matrix that maximizes the information rate given in Equation (2). This optimization problem can be solved efficiently using the Blahut-Arimoto algorithm [8].

III. Proposed P300 Speller Model

The contribution of the channel model presented in this paper is that it takes into account the memory in the system, which is induced by refractory effects.

A. Channel Model

We model the ERP elicitation process as communication through an indecomposable ISI FSC followed by a noisy memoryless channel (Figure 3). Note that a memoryless channel corresponds to an FSC with one state. Increasing the number of states in the FSC allows us to account for the refractory effect as the memory in the channel .

1) States of the channel: The FSC models the channel memory. It maps the input X^n to an intermediate output Z^n. It is possible to design channels with multiple levels of memory. For the purpose of this paper, we consider an FSC with one level of memory, $L = 1$, where the current intermediate output Z_n depends on the current input X_n, and the previous input X_{n-1}. An FSC with $L = 1$ has two states. We define the states as 'Ground' - G and 'Refractory' - R. The FSC input, X_n controls the channel state transitions and the intermediate output, Z_n. In state G, if $X_n = 1$, the channel transitions to state R.

In state R, there are two possible models. In the *input-sensitive* model, if $X_n = 1$, the channel stays in R and output $Z_n = 0$; this represents the reduced ability to elicit a P300 ERP response with consecutive target character presentations due to the refractory effect. The channel returns to G only when $X_n = 0$. In contrast, in the *input-insensitive* model, the return to G from R does not depend on the input. In this scenario, at the refractory state, the channel transitions back to G for any input X_n. A representation of the *input-sensitive* FSC is shown in Figure 4.

The appropriate model for the P300 speller is an open question, and it could be a combination of the input-sensitive and input-insensitive models. This type of a hybrid channel is relatively complex, it is no longer an ISI channel. Research has suggested the P300 amplitudes for a sequence of the form '01' is higher than the amplitudes for sequences of the form '11' [17]. This would suggest that a sequence of the form '111' should generate P300 ERP responses with low amplitudes [11]. Consequently, we focus on the *input-sensitive* refractory period model, where we assume an FSC with one level of memory.

2) Noise: In our model, the noise represents the error in the detection and classification stage. The noise is modeled by a memoryless channel, which maps the intermediate FSC output Z^n to the observed channel output Y^n. We model the memoryless channel with either a binary symmetric channel (BSC), with flip error probability ϵ, or as an additive white Gaussian noise (AWGN) channel with binary input and noise power, σ^2.

B. Codebook Design

Consider a random codebook $\mathbf{C}$ with infinite length. In a memoryless channel, it is well known that an i.i.d. random codebook achieves capacity in the limit of large block lengths. The row and columns of the random codebook for a memoryless channel are independent. The columns of the codebook are related as

$$P(\mathbf{C}_n|\mathbf{C}_{-\infty}^{n-1}) = P(\mathbf{C}_n),$$

where $\mathbf{C}_n$ is the nth column of the codebook, corresponding to the flash group at time n.

For a channel with memory, the rows of the codebook can be independent. However, due to the memory in the channel, a codebook where the columns are independent will not achieve capacity of the channel in general. In our codebook design process, the codewords are drawn from the distribution induced by the r-th order Markov source that maximizes the information rate, P_r^*, as described in Section II-B. The columns of the codebook have the following structure,

$$P(\mathbf{C}_n|\mathbf{C}_{-\infty}^{n-1}) = P(\mathbf{C}_n|\mathbf{C}_{n-r}^{n-1}).$$

The transition matrix depends on both the memory of the channel and the noise in the channel. In this way, we design a

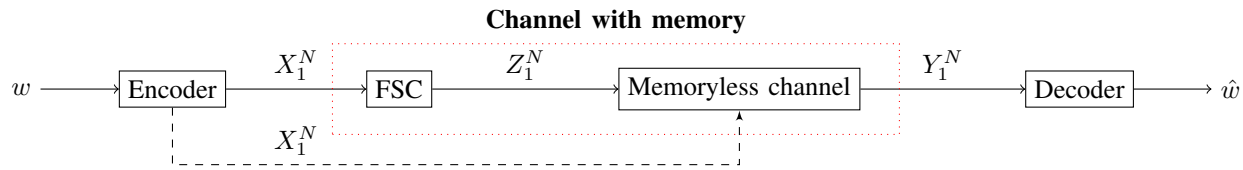


Fig. 3: A channel with memory is modeled as a cascade of a noiseless finite state channel (FSC) and a noisy memoryless channel. A message, w, is encoded with a codeword, X_1^N which is transmitted through the noisy channel. The output sequence, Y_1^N, is observed at the receiver and is used estimate the message, $\hat{w}$. In our proposed model, we model a channel with memory as a cascade of a finite state channel (FSC) with an intermediate output, Z_1^N, and a memoryless channel. The input to the memoryless channel only is represented by the dotted line, and it bypasses the FSC with $X_1^N = Z_1^N$.

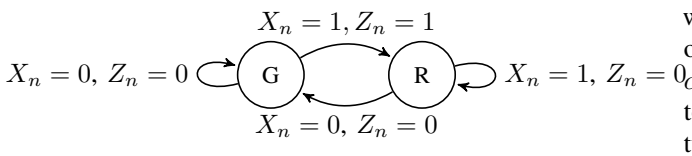


Fig. 4: *Input-sensitive* finite state channel. X_n and Z_n denote the channel input and the intermediate output of the FSC, respectively. G and R denote the ground and refractory states, respectively.

codebook that is optimized for the channel parameters, which we refer to as a memory-based codebook.

IV. Results

Using the BCI model shown in Figure 3, we perform numerical simulations to compare the performances of four different codebooks: RCP, CBP, D10, and our memory-based codebook (MBC). We estimate accuracy as a function of the channel noise parameter for both a memoryless channel and a channel with memory, illustrated in Figure 4.

A. Information Rates with a First Order Markov Source

In order to design the codebook, we first need to find a distribution on the input that maximizes the information rate for a given noise model. For a first order Markov process and BSC noise model, the transition matrix can be represented as,

$$P_{1,(B)} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix},$$

where $\alpha = P(X_n = 1|X_{n-1} = 0)$ and $\beta = P(X_n = 0|X_{n-1} = 1)$.

It can be difficult to analytically compute the information rate for all values of α and β. The mutual information corresponding to the boundary cases $\beta = 1$ or $\alpha = 1$ and a noiseless channel ($\epsilon = 0$) can be computed analytically. The case $\beta = 1$ corresponds to a hard constraint that the channel input cannot a sequence with have two consecutive ones in a row, i.e. '11, and the corresponding information is given by

$$\begin{aligned}\mathcal{I}(P_{1,(B)}, \beta = 1, \epsilon = 0) &= P(X_n = 0)H_b(\alpha) \\ &= \frac{H_b(\alpha)}{\alpha + 1}.\end{aligned} \tag{4}$$

where H_b is the binary entropy, and ϵ is the flip probability of the BSC. The maximum for this equation is obtained at $\alpha = \frac{1}{2}(3 - \sqrt{5})$. Alternatively, the case $\alpha = 1$ corresponds to the contraint on the channel input where it cannot have two '0's in a row. The information rate is given by

$$\begin{aligned}\mathcal{I}(P_{1,(B)}, \alpha = 1, \epsilon = 0) &= P(X_n = 1)H_b(\beta) \\ &= \frac{H_b(\beta)}{1 + \beta}.\end{aligned} \tag{5}$$

The value of β that achieves the maximum is given by $\beta = \frac{1}{2}(3 - \sqrt{5})$. Therefore, this channel has two possible input distributions that maximize the information rate.

For general α, β, and ϵ, we compute the information rate numerically using [8, Equation (11)]. Figure 5 shows the results of the grid search for the binary symmetric channel at low noise ($\epsilon = 10^{-7}$). The numerical results demonstrate that the mutual information rate has two maxima. These occur near the maximizers of the boundary cases for $\epsilon = 0$.

We chose the transition matrix that would be a better fit for our application, where $\beta \approx 1$. Since we are designing the codewords to work around the memory in the channel, choosing $\beta \approx 1$ leads to codewords where the character is not flashed succesively.

The transition matrix obtained for the AWGN channel, $P_{1,(G)}$ is similar to the matrix for the BSC.

We can then design the codebook by treating each row of the codebook as an independent first order Markov source with distribution induced by P_1^*. An instantiation of our memory-based codebook for $\alpha = 0.38$ and $\beta = 0.99$ is shown in Figure 2d. It can be observed that the same character is almost never flashed twice.

B. P300 Speller Simulations

In the simulations run, we selected one of 36 characters uniformly as the target character. The codeword associated with that character is transmitted across the channel, for both the memoryless channel and channel with memory. At the decoder, the received sequence is used to estimate the the target character. The accuracy is the percentage of characters that were correctly estimated over 1000 independent trials.

Figure 6 shows results for a memoryless channel. With a memoryless channel assumption, represented by the dotted line in Figure 3, the performance of a decoder depends

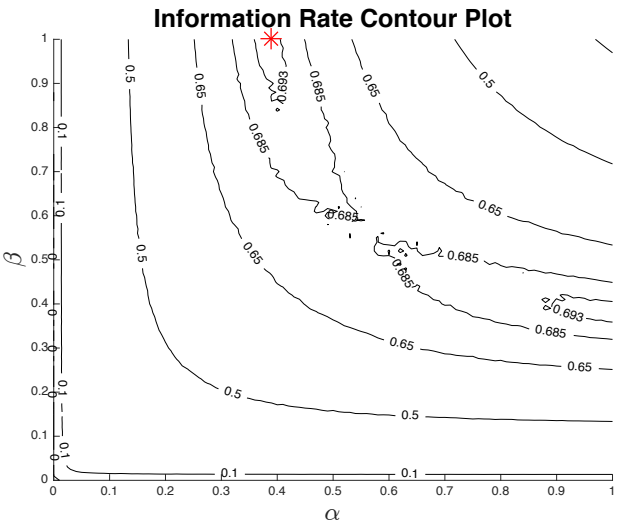


Fig. 5: Contour plot obtained by the grid search for maximum information rate achieved by a first order Markov source input to the channel described in Figure 4. The x-axis α and the y-axis β are the state transition probabilities. The marked point is associated with P_1^*

primarily on the Hamming weight of the codewords, and the Hamming distances between codewords. The codewords from the D10 codebook are dense and the codebook also has the highest minimum Hamming distance. Consequently, we observe an improvement in accuracy from CBP $<$ RCP $<$ MBC $<$ D10.

We analyzed performance when we include memory in a channel, by using an FSC-MC cascade. The results with an optimum decoder, where we account for the memory in the channel during the decoding process are shown in Figure 7. The accuracy improves from CBP<RCP<D10<MBC. However, most P300 speller systems don't account for memory during decoding. The results with a memoryless decoder are shown in Figure 8. There is a a sharp drop in the performance of D10 codebook, due to the multiple characters being flashed successively, which increases refractory effects.

V. Discussion

From Figures 6 and 8, we can observe that the performance of the D10 codebook drops when we use a channel with memory (and a memoryless decoder). This lines up well with current observations that in real-time tests, D10 performed equal to or worse than the RCP codebook [11].

The performance of the memory-based codebook is maintained regardless of the type of channel or decoding used. This can be attributed to the fact that the memory-based codebook is designed around the memory constraints of the channel. A similar observation can be made for the CBP. However, we can also observe that a longer TTI does not seem to improve the performance of the CBP codebook due to the sparsity of the codewords. This points to the fact that there is a trade-off in choosing the memory of channel model, since higher memory corresponds to lower TTI in codewords which in turn leads to sparser codewords.

Finally, we have also shown that when we account for the channel memory in decoding, we see an improvement in the performance of other codebooks too. Most P300 speller systems do not account for the memory in the system due to refractory effects. In comparison, with our new approach of modeling of the channel memory, there could potentially be an advantage even for systems using other codebooks.

VI. Conclusions and Future Work

We have provided an alternative theoretical framework to analyze BCI-based communication. The results in this paper provide a first step in the design of a codebook for the P300 speller based on information theoretic analysis of finite-state channels with memory. These preliminary results also provide evidence that accounting for refractory effects during simulations produces results that better reflect real-time performance trends, especially due to the drop in performance of the D10 codebook relative to the RCP codebook in the presence of memory.

There is empirical evidence that having higher TTIs improves the amplitude of the observed P300 response [6]. For future work, we will be studying more complex channel models with higher levels of memory to more accurately represent the BCI channel, as a channel with one level of memory might be too simplistic. Moreover, we have assumed that the intermediate output when in the refractory state is always $Z_n = 0$. This need not be true always as Z_n could have a non-trivial value.

Furthermore, our approach requires verification with EEG data and validation with real-time implementation in healthy and locked-in individuals.

References

[1] J. Wolpaw, N. Birbaumer, D. McFarland, G. Pfurtscheller, and T. Vaughan, "Brain-computer interfaces for communication and control," *Clinical Neurophysiology*, vol. 113, no. 6, pp. 767–91, 2002.

[2] E. W. Sellers, D. J. Krusienski, D. J. McFarland, T. M. Vaughan, and J. R. Wolpaw, "A P300 event-related potential brain–computer interface (bci): The effects of matrix size and inter stimulus interval on performance," *Biological psychology*, vol. 73, no. 3, pp. 242–252, 2006.

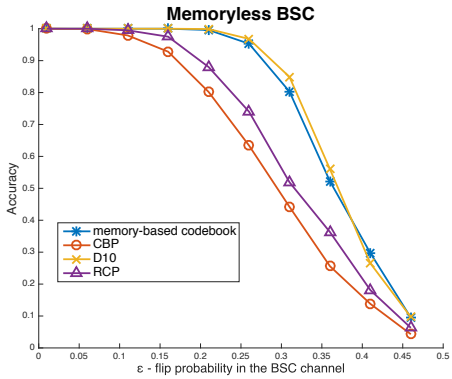
[3] B. O. Mainsah, L. M. Collins, K. A. Colwell, E. W. Sellers, D. B. Ryan, K. Caves, and C. S. Throckmorton, "Increasing BCI communication rates with dynamic stopping towards more practical use: an ALS study," *Journal of Neural Engineering*, vol. 12, no. 1, p. 016013, 2015.

[4] L. A. Farwell and E. Donchin, "Talking off the top of your head: Toward a mental prosthesis utilizing event-related brain potentials," *Electroencephalogr Clin Neurophysiol*, vol. 70, no. 6, pp. 510–523, 1988.

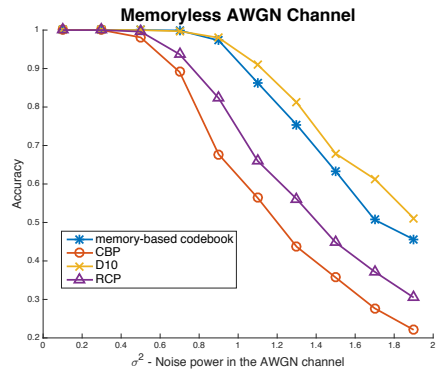
[5] S. Sutton, M. Braren, J. Zubin, and E. R. John, "Evoked-potential correlates of stimulus uncertainty," *Science*, vol. 150, no. 3700, pp. 1187–8, 1965.

[6] J. Jin, E. W. Sellers, and X. Wang, "Targeting an efficient target-to-target interval for P300 speller brain–computer interfaces," *Medical & biological engineering & computing*, vol. 50, no. 3, pp. 289–296, 2012.

[7] S. Martens, N. Hill, J. Farquhar *et al.*, "Overlap and refractory effects in a brain computer interface speller based on the visual P300 event-related potential," *Journal of neural engineering*, vol. 6, no. 2, p. 026003, 2009.

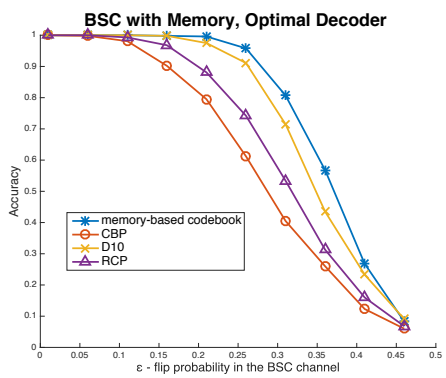


(a)

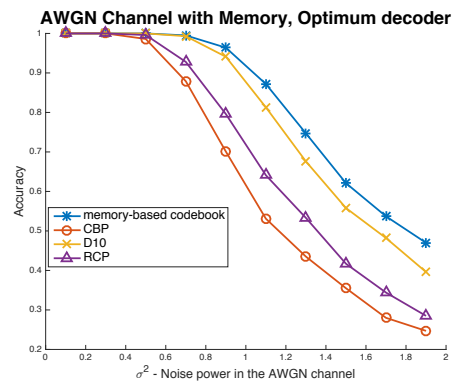


(b)

Fig. 6: Performance in memoryless channels: Accuracy as a function of channel parameter ϵ is represented for a memoryless BSC and σ^2 for an AWGN channel. The D10 codebook performs the best in the memoryless channel since it is optimized for such channels. The memory-based codebook also performs comparitively well.

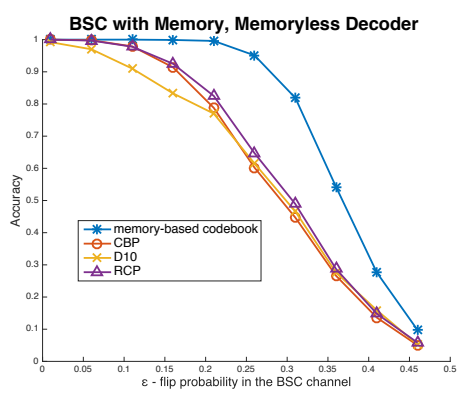


(a)

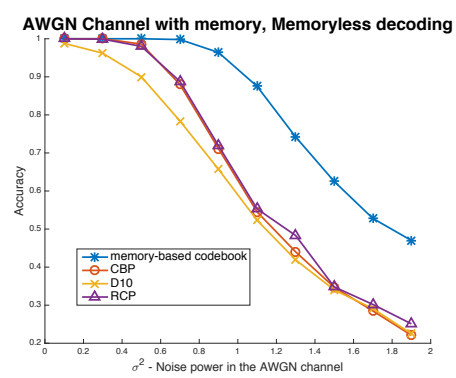


(b)

Fig. 7: Performance in BSC and AWGN Channels using a channel with memory and an optimum decoder: The memory-based codebook performs better than other codebooks in the presence of channel memory.



(a)



(b)

Fig. 8: Performance in a channel with memory with a memoryless decoder: The memory-based codebook maintains performance regardless of the type of decoder or channel.

[8] A. Kavcic, "On the capacity of markov sources over noisy channels," in *Global Telecommunications Conference, 2001. GLOBECOM '01. IEEE*, vol. 5, 2001, pp. 2997–3001.

[9] C. Omar, A. Akce, M. Johnson, T. Bretl, R. Ma, E. Maclin, M. Mc-

Cormick, and T. Coleman, "A feedback information-theoretic approach to the design of brain-computer interfaces," *International Journal of Human-computer Interaction*, vol. 27, pp. 5–23, 2011.

[10] G. Townsend, B. K. LaPallo, C. B. Boulay, D. J. Krusienski, G. E. Frye, C. K. Hauser, N. E. Schwartz, T. M. Vaughan, J. R. Wolpaw, and E. W. Sellers, "A novel P300-based brain-computer interface stimulus presentation paradigm: Moving beyond rows and columns," *Clin Neurophysiol*, vol. 121, no. 7, pp. 1109–20, 2010.

[11] J. Hill, J. Farquhar, S. Martens, F. Biessmann, and B. Schlkopf, "Effects of stimulus type and of error-correcting code design on BCI speller performance," in *Advances in Neural Information Processing Systems 21*. Curran Associates, Inc., 2009, pp. 665–672.

[12] G. Cuntai, M. Thulasidas, and W. Jiankang, "High performance P300 speller for brain-computer interface," in *Biomedical Circuits and Systems*, Dec 2004.

[13] R. Fazel-Rezai, B. Z. Allison, C. Guger, E. W. Sellers, S. C. Kleih, and A. Kübler, "P300 brain computer interface: current challenges and emerging trends," *Frontiers in Neuroengineering*, vol. 5, p. 14, 2012.

[14] J. Geuze, J. D. Farquhar, and P. Desain, "Dense codes at high speeds: Varying stimulus properties to improve visual speller performance," *Journal of neural engineering*, vol. 9, no. 1, p. 016009, 2012.

[15] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.

[16] H. D. Pfister, J. B. Soriaga, and P. H. Siegel, "On the achievable information rates of finite state isi channels," in *Global Telecommunications Conference, 2001. GLOBECOM '01. IEEE*, vol. 5, 2001, pp. 2992–2996 vol.5.

[17] J. Polich, "P300, probability, and interstimulus interval," *Psychophysiology*, vol. 27, no. 4, pp. 396–403, 1990.